The following supplement accompanies the article

The paradox of the plankton: species competition and nutrient feedback sustain phytoplankton diversity

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Supplement. Model parameters and methods used to verify chaos

Model default parameters. The model is applied for 5 phytoplankton species, P_i, competing for 5 resources, N_j, in a well-mixed box following Huisman & Weissing (1999). The default parameters are listed in Table S1. The model Eqs. (1) to (3) in the main article are integrated forward in time using a 4th order Runge-Kutta scheme with a time step of 0.001 d.

Verification of chaos. The emergence of chaos is identified through 3 different approaches: trajectories in phase space where each dimension represents species abundance (illustrated in Fig. 1, right panels, in the main article), Lyapunov Exponent and 0-1 Test for Chaos; these latter 2 approaches are now described in more detail.

The Lyapunov exponent. To identify whether chaos is occurring (as suggested by the phase trajectories), the sensitivity to initial conditions is revealed by estimating the maximal Lyapunov exponent, λ_{max}, which is a measure of the rate at which 2 trajectories diverge over time *t*:

$$\lambda_{max} = \lim_{t\to\infty} \frac{1}{t} \ln\left(\frac{|x(t) - x_\varepsilon(t)|}{|x(0) - x_\varepsilon(0)|}\right), \qquad \text{(S1)}$$

where $x(t)$ and $x_\varepsilon(t)$ are 2 'arbitrary' trajectories starting at a small distance between them (lim: taking the limit of). Negative λ_{max} indicates the convergence of the time series to a steady state and λ_{max}= 0 indicates convergence to 'regular dynamics', i.e. a periodic or quasi-periodic regime. Positive λ_{max} represents an exponential growth in the separation of trajectories and indicates chaos.

To estimate λ_{max} the TISEAN software package is applied (Kantz 1994, Hegger et al. 1999).

The accuracy of the diagnosed λ_{max} is highly sensitive to the length of the time series, as well as the time step and the sampling interval, τ, used for its generation. The analysed time series covered 20 000 d and was sampled with τ = 0.1 d, which, when repeated for the classical Lorenz system, gives a relatively accurate prediction for λ_{max}. The obtained λ_{max} range from 0.007 to 0.035 (Fig. S1), with their small magnitude indicating weak chaos.

The 0-1 Test for Chaos. For a more efficient identification of chaos, we have applied the 0-1 Test for Chaos, a binary test that distinguishes regular from chaotic dynamics (Gottwald &

Melbourne 2009). A statistical characteristic of a very long time series, called K_C, will approach a value of 1 for any value of c if the series is chaotic, and value of 0 if it is regular (where the arbitrary constant $c \in (0,\pi)$; Fig. S2). In simulations with a limited length of time series, the test indicates chaotic dynamics for all values of c only if the series is strongly chaotic. In the case of weak chaos, a longer data series is required. For the data series used in the study, chaos manifests itself initially in a smaller range of values of c, which broadens when a longer time series is analysed (Fig. S2c). Thus, in order for weak chaos to be detected for all values of c, the data series covering at least 10^8 d is needed. For computation efficiency, we generated time series for 50 000 d, and considered the system chaotic when chaos is indicated at the low values of the arbitrary parameter, $c \in (0.2, 0.8)$.

Table S1. Default parameter settings for the model (Huisman & Weissing 1999)

Parameter name	**Values**
Initial concentration of species i, P_i	$P_i = 0.1 + \frac{i}{100}$
Supply concentration of resource j, S_j	$[S_j] = \begin{pmatrix} 6 \\ 10 \\ 14 \\ 4 \\ 9 \end{pmatrix}$
Initial concentration of resource j, N_j	$N_j = S_j$
System's turnover rate, D	0.25 d^{-1}
Maximum phytoplankton growth rate, r_i	1.0 d^{-1}
Mortality rate, m_i	0.25 d^{-1}
Half-saturation coefficient of species i for resource j, K_{ji}	$[K_{ji}] = \begin{pmatrix} 0.39 & 0.34 & 0.30 & 0.24 & 0.23 \\ 0.22 & 0.39 & 0.34 & 0.30 & 0.27 \\ 0.27 & 0.22 & 0.39 & 0.34 & 0.30 \\ 0.30 & 0.24 & 0.22 & 0.39 & 0.34 \\ 0.34 & 0.30 & 0.22 & 0.20 & 0.39 \end{pmatrix}$
Cell quota of species i for resource j, Q_{ji}	$[Q_{ji}] = \begin{pmatrix} 0.04 & 0.04 & 0.07 & 0.04 & 0.04 \\ 0.08 & 0.08 & 0.08 & 0.10 & 0.08 \\ 0.10 & 0.10 & 0.10 & 0.10 & 0.14 \\ 0.05 & 0.03 & 0.03 & 0.03 & 0.03 \\ 0.07 & 0.09 & 0.07 & 0.07 & 0.07 \end{pmatrix}$

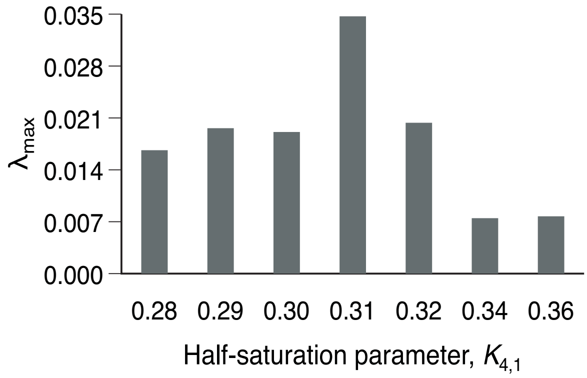

Fig. S1. Estimation of the maximal Lyapunov exponent, λ_{max}, for the chaotic system generated with varying half-saturation coefficient, $K_{4,1}$

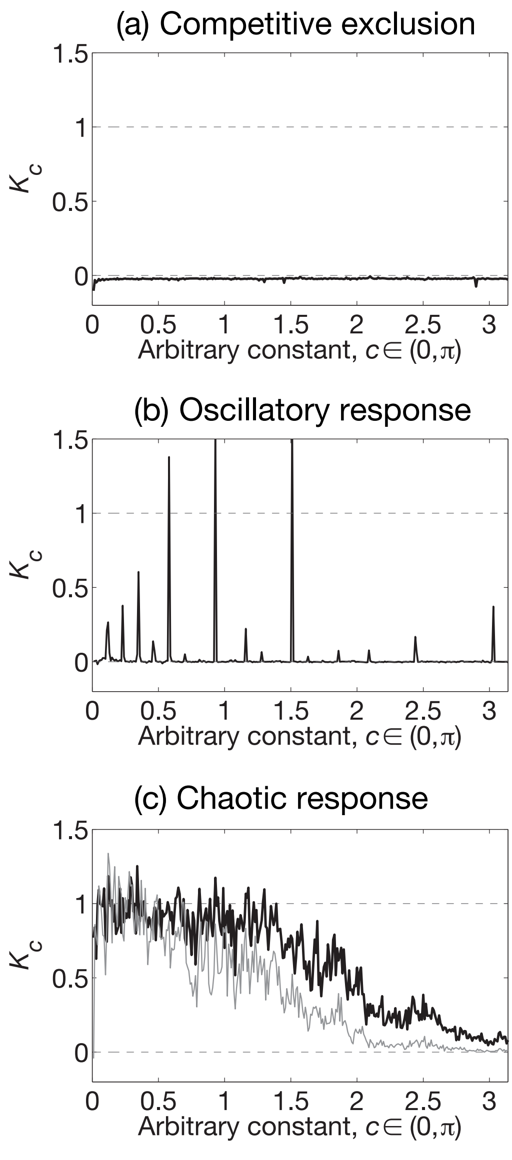

Fig. S2. The 0-1 Test for Chaos (Gottwald & Melbourne 2004, 2009) analysis of different characters for the phytoplankton community responses represented for (a) competitive exclusion, (b) oscillations, and (c) chaos from Fig. 1. The time series of species abundance used for the analysis is generated for 50 000 d. The grey line in (c) represents the output of the 0-1 Test for the time series of 10 000 d

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