Supplement 2. List of models tested for each month

Model number	Model formula
1	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_0$
2	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_1 sst$
3	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_1 eke$
4	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_1 mean\_sst\_grad$
5	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_1 sst + \beta_2 eke$
6	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_1 sst + \beta_2 mean\_sst\_grad$
7	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_1 eke + \beta_2 mean\_sst\_grad$
8	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_1 sst + \beta_2 eke + \beta_3 mean\_sst\_grad$
9	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_{1,t} sst$
10	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_{1,t} eke$
11	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_{1,t} mean\_sst\_grad$
12	$\log\mathrm{MI}_{s,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_{1,t} sst + \beta_{2,t} eke$
13	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_{1,t} sst + \beta_{2,t} mean\_sst\_grad$
14	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_{1,t} eke + \beta_{2,t} mean\_sst\_grad$
15	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_{1,t} sst + \beta_{2,t} eke + \beta_{3,t} mean\_sst\_grad$
16	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{s,t} + \beta_0 + \eta_c$
17	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_1 sst + \eta_c$
18	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_1 eke + \eta_c$
19	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_1 mean\_sst\_grad + \eta_c$
20	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_1 sst + \beta_2 eke + \eta_c$
21	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_1 sst + \beta_2 mean\_sst\_grad + \eta_c$
22	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_1 eke + \beta_2 mean\_sst\_grad + \eta_c$
23	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_1 sst + \beta_2 eke + \beta_3 mean\_sst\_grad + \eta_c$
24	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_{1,t} sst + \eta_c$
25	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_{1,t} eke + \eta_c$
26	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_{1,t} mean\_sst\_grad + \eta_c$
27	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_{1,t} sst + \beta_{2,t} eke + \eta_c$
28	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_{1,t} sst + \beta_{2,t} mean\_sst\_grad + \eta_c$
29	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_{1,t} eke + \beta_{2,t} mean\_sst\_grad + \eta_c$
30	$\log\mathrm{MI}_{c,t} \sim \gamma \times SP_{c,t} + \beta_{0,t} + \beta_{1,t} sst + \beta_{2,t} eke + \beta_{3,t} mean\_sst\_grad + \eta_c$

Supplement 3: Variance partitioning

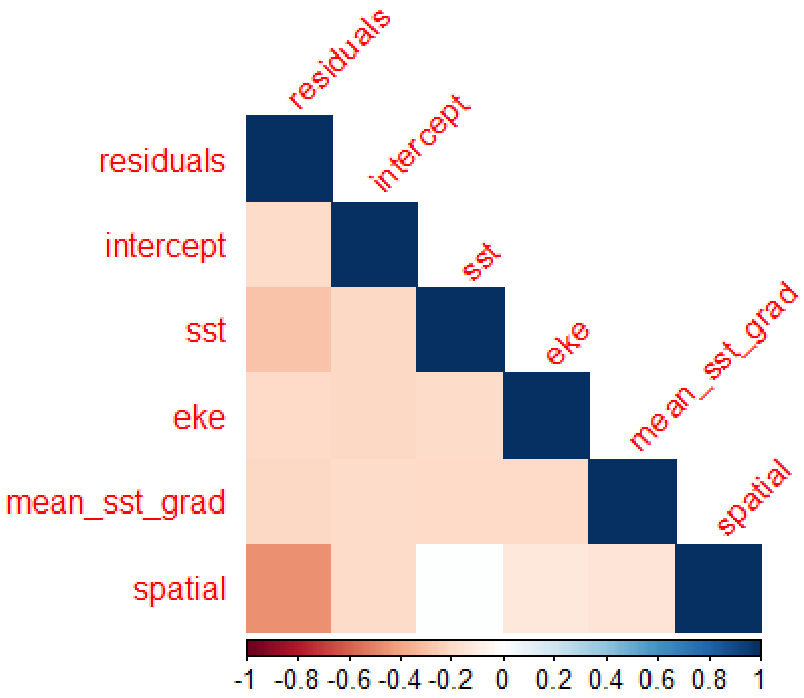
This is an example of how the variance partitioning computation, for the model of January. It was identically applied to the next monthly models and corresponding data frames, from February to December.

Compute the covariance matrix of all model components

First, we calculate the covariance matrix of all random effects: residuals, gaussian random field, year varying intercept and coefficient of covariates. There are three covariates (sst, eke and mean_sst_grad) so that there is in total 6 random effects.

```
sigma2 <- 1 / inla.hyperpar.sample(1e5,
                                   mod,
                                   intern = FALSE,
                                   improve.marginals = TRUE
                                   )
### diagonal matrix with standard deviation
D <- diag(sqrt(apply(sigma2, 2, median)))

### correlation matrix
R <- cor(sigma2)

### write covariance matrix as D R D with D diagonal matrix and R correlati
on matrix
sigma2 <- D %*% R %*% D
```

inla.hyperpar.sample produce samples from the approximated joint posterior for the hyperparameters of our model `mod`. We chose to use median and not mean values because of asymmetric posteriors. Finally, `sigma2` was the covariance matrix of all hyperparameters of the model. Covariances between pairs of hyperparameters can be visualized through the following correlation matrix:



It shows that covariances were all negative, and that the strongest covariance was between the residuals and the spatial field.

Formula (11) from Johnson et al. (2014)

From this covariance matrix, we can compute all variance components. For any random effect l, the mean random effect variance σ_l^2 is:

$$\sigma_l^2 = Tr(Z\sum Z')/n$$

Where n the number of observations, Z is the design matrix for all random effects and Z' the transpose of Z. Z contains n rows and one column per random effect.

We mobilized this formula to compute the variance taken into account by each random effect, adapting the design matrix Z.

First, to compute the total mean variance of the model, we defined Z with all random effects' components:

```
# Design matrix
n <- nrow(subset(data, year != 2019))
Z <- cbind(rep(1, n), # residual
           rep(1, n), # yearly intercept
           subset(data, year != 2019)$sst,
           subset(data, year != 2019)$eke,
           subset(data, year != 2019)$mean_sst_grad,
           rep(1, n) # spatial
          )
```

Where `data` is the dataset corresponding to January. 2019 was excluded as no mortality data was given for the out-of-sample cross validation, thus it wasn't used to infer these random effects. Then we computed the total variance based on formula (11):

```
omega <- Z %*% sigma2 %*% t(Z) # computationally demanding part, can be dec
omposed
# formula (11) of Johnson et al. (2014)
sigma2_tot <- sum(diag(omega)) / n
# remove objects
rm(omega, Z)
```

Note that, for the computation of `sigma2_tot`, all covariance terms are included by the mean of the covariance matrix and the complete design matrix.

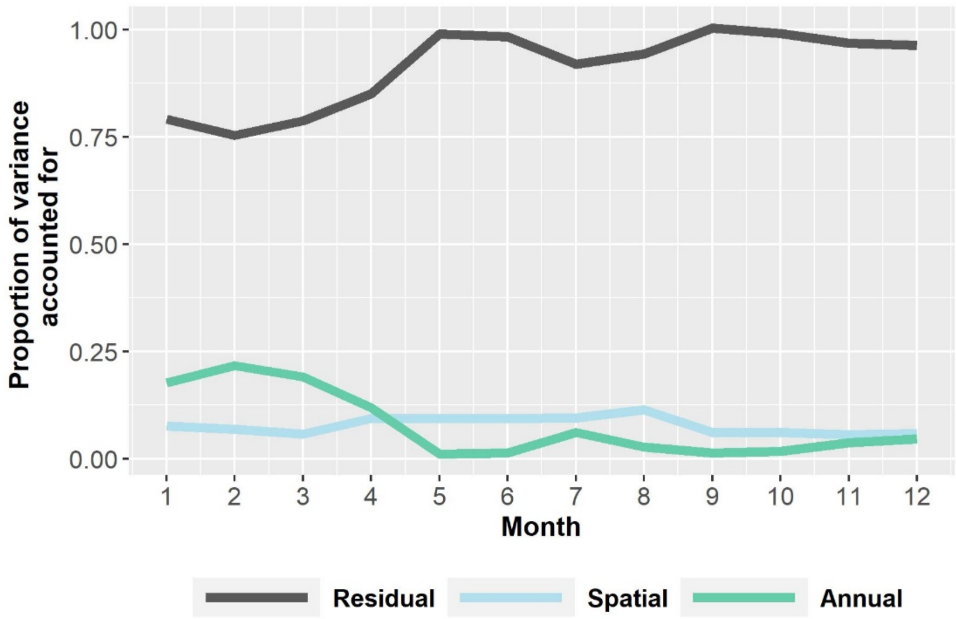
Then, this was used to compute the two observation-level mean variance components, i.e. residuals and spatial (gaussian random field).

```
# residual variance: variance not accounted for
residual_var <- sigma2[1, 1] / sigma2_tot

# spatial variance: variance accounted for by the gaussian random field
spatial_var <- sigma2[6, 6] / sigma2_tot
```

Finally, for yearly random slopes, there is another level of variation: year. This imply a dependence between the variance taken into account by each covariate's effect and observations, i.e. values taken by covariates in a given cell and a given year. There are also between these random effects covariance terms that needs to be taken into account by the mean of the design matrix. We computed the mean variance taken into account by these 4 year-level random effects:

```
# Design matrix
Z <- cbind(rep(0, n), # residual
           rep(1, n), # yearly intercept
           subset(data, year != 2019)$sst,
           subset(data, year != 2019)$eke,
           subset(data, year != 2019)$mean_sst_grad,
           rep(0, n) # spatial
)
omega <- Z %*% sigma2 %*% t(Z)
# formula (11) of Johnson et al. (2014)
sigma2_year <- sum(diag(omega)) / n
# remove objects
rm(omega, Z)

# Total variance accounted for by year-level random effects
year_var <- sigma2_year / sigma2_tot
```

`residual_var`, `spatial_var` and `year_var` were the three input values (computed for the 12 monthly models) used for Fig.6 of the article, shown below.



Note that, if covariance terms were included in the computation of `sigma2_tot` and `sigma2_year` (with the design matrix), there were not included for the computation of each covariate's contribution, nor for `spatial_var` and `residual_var`. Consequently, for some months, `residual_var + spatial_var + year_var` and `intercept_var + sst_var + eke_var + mean_sst_grad_var` can be superior to one. It is especially noteworthy for the former, for instance in June or September. This is due to a strong negative covariance between spatial and residual terms (see the correlation matrix of `sigma2` above).

Literature cited

Johnson PC (2014) Extension of Nakagawa & Schielzeth's R2GLMM to random slopes models. *Methods in Ecology and Evolution*, 5(9):944–946