The following supplements accompany the article

Improved abundance and trend estimates for sperm whales in the eastern North Pacific from Bayesian hierarchical modeling

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Supplement 1. Sampling variance for the number of groups of sperm whales *Physeter macrocephalus* sighted in each stratum-year, for the generalized Poisson distribution (Famoye 1993, Famoye et al. 2004).

For the generalized Poisson distribution, the mean is μ_{gjt} and the variance is $\sigma^2_{gjt} = \mu_{gjt}(1 + \alpha_{gjt}\mu_{gjt})^2$, where α_{gjt} is an overdispersion parameter and subscripts *g* = group-size class (small = 1; large = 2); *j* = stratum; *t* = year.

Rearranging the equation for α_{gjt}, and calling σ^2_{gjt}/μ_{gjt} the variance inflation factor v_{gjt}, $\alpha_{gjt} = \frac{\sqrt{v_{gjt}}-1}{\mu_{gjt}}$.

We used the *g*-specific averages of the $\hat{v}_{gjt}$ (i.e., $\overline{v_g}$) as the estimate for v_{gjt} in the model, since the sampling process was similar for all years and strata and the $\hat{v}_{gjt}$ did not suggest any obvious pattern with respect to *t* or *j*. We estimated $\overline{v_g}$ using a bootstrapping approach. As described by Barlow & Forney (2007), the total transect effort was divided into segments of approximately 150 km, corresponding to the amount of survey effort conducted in 1 d. Effort segments were sampled with replacement thousands of times, and the number of observed groups n_{gjt} was recorded for each bootstrap sample. The means and variances were calculated from the bootstrap samples, providing the $\hat{v}_{gjt}$. These were then averaged (weighted by n_{gjt}) across the *j,t* to calculate $\overline{v_g}$. For small groups, $\overline{v_1}$ = 1.3 (SE = 0.08), suggesting fairly low overdispersion in the sighting frequencies, but for larger groups, $\overline{v_2}$ = 2.3 (SE = 0.33), implying that sampling variance in the number of groups sighted was slightly more than twice that of a simple Poisson process.

The log likelihood function for the generalized Poisson distribution is:

$$\log\mathcal{L}(\mu_{gjt}|n_{gjt}) = n_{gjt}\left[\log\mu_{gjt} - \log(1 + \alpha_{gjt}\mu_{gjt})\right] + (n_{gjt} - 1)\log(1 + \alpha_{gjt}n_{gjt}) - \frac{\mu_{gjt}(1 + \alpha_{gjt}n_{gjt})}{1 + \alpha_{gjt}\mu_{gjt}} - \log(n_{gjt}!) \quad \text{(S1)}$$

Given the empirical field estimates of $\overline{v_g}$, the log-likelihood then simply varies with modeled estimates of μ_{gjt}, since $\alpha_{gjt} = \frac{\sqrt{\overline{v_g}}-1}{\mu_{gjt}}$.

LITERATURE CITED

Barlow J, Forney KA (2007) Abundance and the population density of cetaceans in the California Current ecosystem. Fish Bull 105:509–526

Famoye F (1993) Restricted generalized Poisson regression model. Comm Stat Theory Methods 22:1335–1354

Famoye F, Wulu JT Jr, Singh KP (2004) On the generalized Poisson regression model with an application to accident data. J Data Sci 2:287–295

Supplement 2. Simple derivation of Horvitz-Thompson estimator for expected number of detected groups, μ.

This estimator is asymptotically unbiased but can be problematic with small samples. In our analysis, we used an alternative approach, for which μ is not conditioned on n (see 'Methods in the main text').

From Marques & Buckland (2004), the conventional covariate-dependent estimator for density D is:

$$D = \frac{1}{2L} \sum_{i=1}^{n} \frac{f(0|\mathbf{z}) \cdot s}{g(0|\mathbf{z})}, \qquad \text{(S2)}$$

where L is the length of transect surveyed, n is the number of groups detected, s is group size, $\mathbf{z}$ is a vector of covariates, and $f(0)$ and $g(0)$ are parameters related to detection probability. Multiply both sides by the expected number of groups to be detected, μ, and isolate one of them:

$$\mu = D \cdot 2L \cdot \frac{\mu}{\sum_{i=1}^{n} s_i f_i(0|\mathbf{z}_i)/g_i(0|\mathbf{z}_i)}. \qquad \text{(S3)}$$

Using the observed number of groups, n, as the estimate for μ on the right-hand side:

$$\mu = D \cdot 2L \cdot \frac{n}{\sum_{i=1}^{n} s_i f_i(0|\mathbf{z}_i)/g_i(0|\mathbf{z}_i)}, \qquad \text{(S4)}$$

or if detection probability is independent of group size, then

$$\mu = D \cdot 2L \cdot \frac{1}{\bar{s}} \cdot \frac{n}{\sum_{i=1}^{n} f_i(0|\mathbf{z}_i)/g_i(0|\mathbf{z}_i)}. \qquad \text{(S5)}$$

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Marques FFC, Buckland ST (2004) Covariate models for the detection function. In: Buckland ST, Anderson DR, Burnham KP, Laake JL, Borchers DL, Thomas L (eds) Advanced distance sampling. Oxford University Press, New York, NY, p 31–47

Supplement 3. Estimating s_{gjt}, the mean group size, by year (t) and stratum (j), for sperm whales *Physeter macrocephalus* of group size > 2 (g = 2).

Greater effort was devoted to estimating sperm whale group sizes in the 3 most recent surveys (2001, 2005, 2008) than in earlier surveys, and group size is known to be underestimated in the earlier surveys for this reason. We estimated true mean group size, s_{2jt}, by treating transformed individual group size observations as generalized Poisson random variables. Observed group sizes, s_{2jti} (Fig. S1), were transformed by subtracting 2, so the data would consist of values ≥ 0. We call the transformed (T) group sizes s^T_{2jti}, such that $s^T_{2jti} \sim gPois(s^T_{2jt}, \alpha)$, where $s^T_{2jt} = \exp(\beta_0 + \beta_1 m + \varepsilon_{jt})$, and where ε_{jt} is the normally distributed random effect with mean 0 and represents stratum-year departures from the overall mean. The overdispersion parameter, α, may be estimated by the model, but model estimates in our analysis were too high (estimated variance was much higher than variance in the data), likely indicating a poor fit of the generalized Poisson model to the data, so we used a value of α = 0.12, which generated variance estimates similar to that of that data. The coefficient β_1 represents the mean difference in observed vs. true group size for early surveys (m = 1) compared to recent surveys (m = 0), i.e. the effect of different survey protocols. The conditional estimate for true mean group size, s_{2jt}, is $\exp(\beta_0 + \varepsilon_{jt})$ + 2. The +2 is for back-transforming.

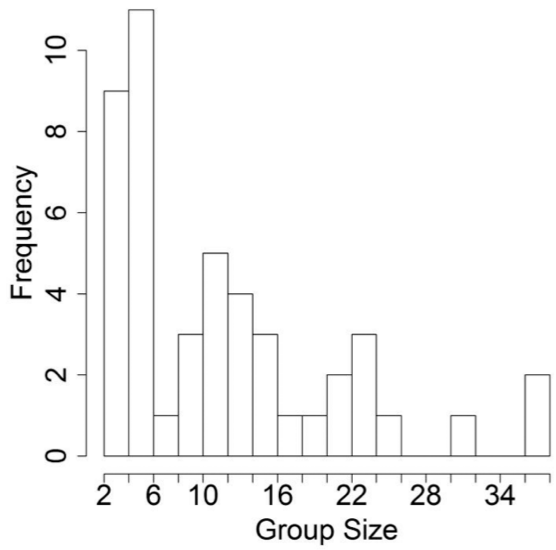

Fig. S1. Sperm whale *Physeter macrocephalus* group sizes > 2

Supplement 4. Model parameters to be estimated

Table S1. Prior distributions for estimated model parameters

Parameter	Prior	Description
Process model		
log(N0)	Unif(4, 10)	Log abundance for year = 0 (1990)
r	Unif(−1, 1)	Instantaneous rate of change for abundance within study area
$\sigma_{process}$	Unif(0, 3)	Standard deviation of process variation in annual abundance
logit($\varphi_{g=1,t}$)	Norm($\mu_{\varphi1}$, $\tau_{\varphi1}$)	Logit proportion of abundance occurring within small groups in each year t
$\mu_{\varphi1}$	Norm(0, 0.00001[a])	Mean parameter for logit($\varphi_{g=1,t}$)
$\tau_{\varphi1}$	$1/(\sigma_{\varphi1})^2$; $\sigma_{\varphi1}$~ Unif(0, 10)	Precision parameter for logit($\varphi_{g=1,t}$)
$u_{g=2,j}$	Unif(0, 100)	Dirichlet parameter for proportion of large-group abundance occurring in each of the J strata
$u_{g=1,j}$	Unif(0, 100)	Dirichlet parameter for proportion of small-group abundance occurring in each of the J strata
Observation model		
$\beta_{esw,\, g=1}$	Unif(−10, 4)	Log of the half-normal scale parameter for detection (effective-strip-width) function, for small groups
$g_b(0)$	See Table 2 in the main text	Beaufort sea state specific estimate of detection probability for animals on the transect line
$\beta_{0,s,\, g=2}$	Unif(−5, 5)	Intercept parameter for transformed group size variable on log scale (large groups)
$\beta_{1,s,\, g=2}$	Unif(−10, 0)	Correction factor for transformed group size for first 3 survey years (large groups)
$\sigma_{s,\, g=2}$	Unif (0, 4)	Standard deviation of variation in mean transformed group size (on log scale) across strata and years (large groups)

[a]Precision = 1/variance